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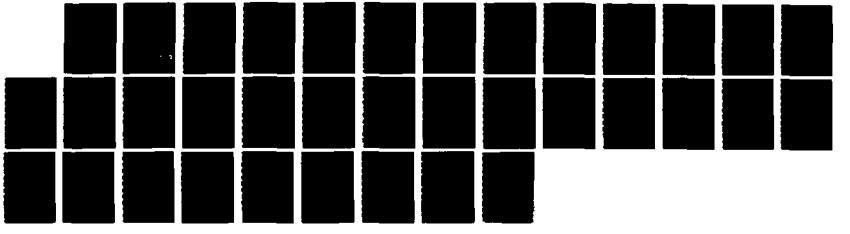

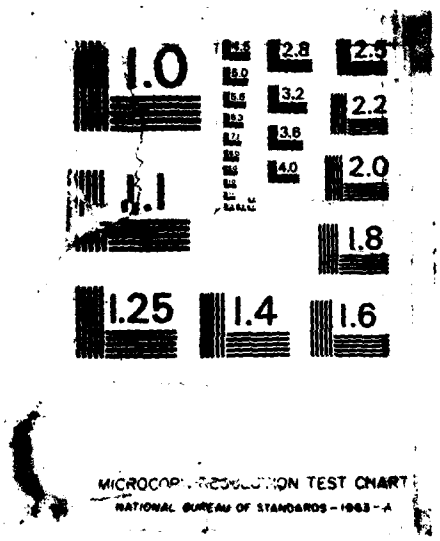
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February 1984

INDIVIDUAL GEOPOTENTIAL COEFFICIENTS OF ORDER 15 AND 30, FROM RESONANT SATELLITE ORBITS

by

D. G. King-Hele
Doreen M. C. Walker

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SUMMARY

The analysis of variations in satellite orbits when they pass through 15th-order resonance (15 revolutions per day) yields values of lumped geopotential harmonics of order 15, and sometimes of order 30. The 15th-order lumped harmonics obtained from 24 such analyses over a wide range of orbital inclinations are used here to determine individual harmonic coefficients of order 15 and degree 15, 16,...35; and the 30th-order lumped harmonics (from eight of the analyses) are used to evaluate individual coefficients of order 30 and degree 30, 32,...40. The new values should be more accurate than any previously obtained. The accuracy of the 15th-order coefficients of degree 15, 16,...23 is equivalent to 1 cm in geoid height, while the 30th-order coefficients of degree 30, 32 and 34 are determined with an accuracy which is equivalent to better than 2 cm in geoid height. The results are used to assess the accuracy of the Goddard Earth Model 10B.

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1 INTRODUCTION

Three years ago we obtained[1] values of individual 15th-order coefficients in the geopotential up to degree 35, from lumped harmonics of order 15 derived by analysis of 23 satellite orbits that passed through 15th-order resonance with the Earth's gravity field. These orbits covered a wide range of inclinations to the equator, but there were no accurate orbits at inclinations between 59° and 74°. The recent orbit analysis[2] of a satellite at 65.8° inclination (1971-10B) should make possible a better evaluation of individual 15th-order coefficients.

Analysis of a satellite orbit that passes slowly through 15th-order resonance also often yields values for lumped harmonics of 30th order, and previously there were results from orbits at enough different inclinations to obtain a preliminary solution[3] for individual coefficients of 30th order. However, there were no results from orbits at inclinations between 59° and 74°. This gap has again been filled by the results from 1971-10B, and greatly improved solutions for individual 30th-order coefficients should emerge.

2 BACKGROUND

The gravitational potential of the Earth is usually expressed as a double infinite series of tesseral harmonics depending on latitude and longitude. The order m of the harmonics expresses the variation with longitude, and a harmonic of order m has m sinusoidal oscillations over 360° of longitude. The degree ℓ of the harmonic (where $\ell \geq m$) governs variations with latitude, which are more complex[4].

If the orbital period of a satellite is such that its successive ground tracks over the Earth are $360^\circ/m$ apart, so that the track repeats after m revolutions, the satellite exhibits mth-order resonance and the perturbations due to harmonics of order m build up day after day to produce quite a large change in some of the orbital elements. This change can be analysed to determine a lumped harmonic of order m, that is a linear sum of individual harmonics of order m and degree ℓ_o, $\ell_o + 2$, $\ell_o + 4$, ..., where $\ell_o = m$ or $m + 1$ (depending on the orbital element being analysed, and whether m is odd or even). By obtaining values of lumped harmonics for many resonant satellites at different inclinations to the equator, it is possible to solve for the individual harmonic coefficients.

A satellite experiencing 15th-order resonance usually has an average height near 500 km - the exact value depends on the inclination, being 470 km for an equatorial orbit and 560 km for a polar orbit. At these heights the effects of atmospheric drag are appreciable: so the contraction of the orbit under the influence of air drag brings it to resonance and slowly draws it through resonance. The lower the drag, the longer the resonance acts, and the better the orbit is for analysis.

The theory of the resonance has been given in Ref 1 and elsewhere, and will not be repeated here; but the notation is outlined in section 3.

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3 NOTATION

The longitude-dependent part of the geopotential at an exterior point (r,θ,λ) is written in normalized form[5] as

$$\frac{\mu}{r}\sum_{\ell=2}^{\infty}\sum_{m=1}^{\ell}\left(\frac{R}{r}\right)^{\ell} P_{\ell}^{m}(\cos\theta)\left\{\bar{C}_{\ell m}\cos m\lambda + \bar{S}_{\ell m}\sin m\lambda\right\}N_{\ell m} \quad , \tag{1}$$

where r is the distance from the Earth's centre, θ is co-latitude, λ is longitude (positive to the east), μ is the gravitational constant for the Earth (398600 km^3/s^2) and R is the Earth's equatorial radius (6378.1 km). The $P_{\ell}^{m}(\cos\theta)$ are the associated Legendre functions of order m and degree ℓ, and $\bar{C}_{\ell m}$ and $\bar{S}_{\ell m}$ are the normalized tesseral harmonic coefficients: only those of order $m = 15$ and $m = 30$ are relevant here. The normalizing factor $N_{\ell m}$ is given by[5]

$$N_{\ell m}^2 = \frac{2(2\ell + 1)(\ell - m)!}{(\ell + m)!} \quad . \tag{2}$$

Resonance is defined by means of the resonance angle Φ, given for 15th-order resonance by

$$\Phi = \omega + M + 15(\Omega - \nu) \quad , \tag{3}$$

where ω is the argument of perigee, M the mean anomaly, Ω the right ascension of the node and ν the sidereal angle. Exact resonance occurs when

$$\dot{\Phi} = \dot{\omega} + \dot{M} + 15(\dot{\Omega} - 360.987) \quad \text{deg/day} \tag{4}$$

is zero, and, in practice, perturbations due to resonance are usually appreciable if $|\dot{\Phi}| < 10$ deg/day.

The rate of change of inclination i caused by a relevant pair of geopotential coefficients, $\bar{C}_{\ell m}$ and $\bar{S}_{\ell m}$, near a resonance may be written[6]

$$\frac{di}{dt} = \frac{n}{\sin i}\left(\frac{R}{a}\right)^{\ell}\bar{F}_{\ell mp}G_{\ell pq}(k\cos i - m)\Re\left[j^{\ell-m+1}(\bar{C}_{\ell m} - j\bar{S}_{\ell m})\exp\{j(\gamma\Phi - q\omega)\}\right] \quad , \tag{5}$$

where $n = \dot{M}$, a is the same major axis, $\bar{F}_{\ell mp}$ is Allan's normalized inclination function[6], $G_{\ell pq}$ is a function of eccentricity e, $\Re$ denotes 'real part of' and $j = \sqrt{-1}$. The indices γ, q, k and p in equation (5) are integers, with γ taking the values 1, 2, 3,... and q the values 0, ±1, ±2, ... For 15th-order resonance, the equations between ℓ, m, k and p are: $m = 15\gamma$; $k = \gamma - q$; $2p = \ell - k$.

The largest terms in equation (5) are nearly always those with $\gamma = 1$, but the $\gamma = 2$ terms are sometimes important too. With $\gamma = 1$ and $\gamma = 2$, the values of m that arise are $m = 15$ and $m = 30$ respectively. For given m, the values of ℓ that arise are those for which $(\ell - k)$ is even; also, of course, $\ell \geq m$. Thus, if the minimum possible value of ℓ is denoted by ℓ_o, where ℓ_o is either m or $(m + 1)$, the values of ℓ that arise are ℓ_o, $\ell_o + 2$, $\ell_o + 4$, ... It is convenient to group these successive

relevant coefficients in the form of a 'lumped harmonic', written as

$$\bar{C}_m^{q,k} = \sum_\ell Q_\ell^{q,k} \bar{C}_{\ell m} \quad , \qquad \bar{S}_m^{q,k} = \sum_\ell Q_\ell^{q,k} \bar{S}_{\ell m} \quad , \tag{6}$$

where the $Q_\ell^{q,k}$ are functions of i that can be taken constant for a particular satellite, $Q_\ell^{q,k} = 1$ when $\ell = \ell_o$, and the summation is for values of ℓ from ℓ_o upwards in steps of 2.

For 15th-order resonance, the rate of change of i may be expressed in terms of the lumped coefficients as[7]

$$\begin{aligned} \frac{di}{dt} = \frac{n}{\sin i}\left(\frac{R}{a}\right)^{15}(15 - \cos i)\Bigg[& \bar{F}_{15,15,7}\left\{\bar{C}_{15}^{0,1} \sin\phi - \bar{S}_{15}^{0,1} \cos\phi\right\} \\ & + 2\left(\frac{R}{a}\right)^{15} \bar{F}_{30,30,14}\left\{\bar{C}_{30}^{0,2} \sin 2\phi - \bar{S}_{30}^{0,2} \cos 2\phi\right\} \\ & + \text{terms in } \frac{(\frac{1}{2}\ell e)^{|q|}}{(|q|)!} \begin{matrix}\cos \\ \sin\end{matrix}(\gamma\phi - q\omega)\Bigg] \quad , \end{aligned} \tag{7}$$

where only the terms in $(\gamma,q) = (1,0)$ and $(2,0)$ are given explicitly, although it is often necessary to take account also of the $(\gamma,q) = (1,1)$ and $(1,-1)$ terms (unless the eccentricity is very small).

The rate of change of eccentricity at 15th-order resonance, produced mainly by the terms with $(\gamma,q) = (1,1)$ and $(1,-1)$, may be written[1]:

$$\begin{aligned} \frac{de}{dt} = \frac{n}{2}\left(\frac{R}{a}\right)^{16}\Bigg[\Bigg[& -17\bar{F}_{16,15,8}\left\{\bar{S}_{15}^{1,0} \sin(\phi - \omega) + \bar{C}_{15}^{1,0} \cos(\phi - \omega)\right\} \\ & + 13\bar{F}_{16,15,7}\left\{\bar{S}_{15}^{-1,2} \sin(\phi + \omega) + \bar{C}_{15}^{-1,2} \cos(\phi + \omega)\right\} \\ & + \text{terms in } \left[\frac{(\frac{1}{2}\ell)^{|q|} e^{|q|-1}}{(|q|)!}\left\{q - \tfrac{1}{2}(k+q)e^2\right\} \begin{matrix}\cos \\ \sin\end{matrix}(\gamma\phi - q\omega)\right]\Bigg] \quad . \end{aligned} \tag{8}$$

Thus analysis of the variation in inclination usually gives values of lumped harmonics with $(q,k) = (0,1)$ and $(0,2)$, while analysis of the variation in eccentricity gives values of lumped harmonics with $(q,k) = (1,0)$ and $(-1,2)$.

4 PROCEDURE

The methods of analysis have been explained previously[1]. In summary, the observational values of inclination are cleared of irrelevant perturbations and fitted using the computer program THROE[8] with an integrated form of the theoretical equation (7), with extra terms when appropriate, to determine values of the lumped coefficients. Similarly the observational values of eccentricity, cleared of perturbations, are fitted with an integrated form of equation (8), with extra terms as necessary. With a few satellites it is useful to make a simultaneous fitting of inclination and eccentricity using the SIMRES program.

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Each fitting yields values of one or more pairs of lumped harmonics, and each lumped harmonic specifies one linear equation between the individual harmonics of odd degree (or those of even degree). If such equations are available from fittings of orbits over a wide range of inclinations, the equations can be solved for individual harmonic coefficients. In practice the effects usually decrease quite rapidly as the degree of the harmonics increases, and the truncation of the solutions at a particular degree is probably not a limiting factor on the accuracy, although it is a matter that calls for careful judgement.

Previously[1], we used results from 23 orbit analyses to determine individual 15th-order coefficients of degree up to 35. For 30th order[3], we offered a tentative solution for individual coefficients of degree 30, 32, ...40, from 7 orbit analyses. The addition of 1971-10B now gives us 24 orbits for determining individual 15th-order coefficients and 8 for 30th-order. Most of the existing analyses are accepted unchanged, but four have been revised. The new and improved analyses are discussed in section 5.

5 NEW OR REVISED VALUES

5.1 1971-10B, Cosmos 394 rocket

The main new results are for the satellite 1971-10B at inclination 65.8°, for which 52 orbits covering the time of 15th-order resonance were determined and analysed by Walker[2]. Values were obtained for six 15th-order lumped coefficients and two 30th-order lumped coefficients, as follows:

$$\left.\begin{aligned} 10^9\bar{C}_{15}^{0,1} &= -0.7 \pm 4.1 & 10^9\bar{S}_{15}^{0,1} &= 2.4 \pm 3.9 \\ 10^9\bar{C}_{15}^{1,0} &= 35.1 \pm 11.7 & 10^9\bar{S}_{15}^{1,0} &= -13.8 \pm 11.0 \\ 10^9\bar{C}_{15}^{-1,2} &= -20.0 \pm 10.7 & 10^9\bar{S}_{15}^{-1,2} &= -18.9 \pm 10.3 \\ 10^9\bar{C}_{30}^{0,2} &= -54 \pm 27 & 10^9\bar{S}_{30}^{0,2} &= 59 \pm 40 \end{aligned}\right\} \quad (9)$$

The first pair of coefficients may seen to be ill-defined, but in fact the standard deviations are small: it so happens that both coefficients are very small at this inclination. Apart from the first pair of coefficients, which have standard deviations similar to those for 1970-87A, these values are much more accurate than any previously determined for a satellite with inclination between 59° and 74°, and they have a major effect on the accuracy and reliability of the final solutions.

5.2 1970-87A, Cosmos 373

For this satellite, at inclination 62.9°, there were only 24 orbits, all at dates after resonance. Previously[1], we used a fitting which omitted the last of the 24 points. After further examination, we decided that the omission of the last point was not really justified, and we have returned to the fitting with all 24 points. Also the atmospheric rotation rate was altered from $\Lambda = 0.8$ to $\Lambda = 0.9$ rev/day, in conformity with Ref 9. The new values for the lumped coefficients are:

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$$10^9\bar{C}^{0,1}_{15} = -5.4 \pm 3.6 \qquad 10^9\bar{S}^{0,1}_{15} = -31.4 \pm 2.8 \quad . \tag{10}$$

The previous values were -5.3 and -32.8 respectively, with slightly lower standard deviations through the omission of the last point. In the solutions it was found necessary to increase the standard deviation of the $\bar{S}$ coefficient by a factor of 4.

5.3 1977-12B, Tansei 3 rocket

This satellite, at inclination 65.5°, was of rather high drag and cannot be expected to give accurate values. Our previous results, based on analysis of US Navy orbits, were not very good and were included only because of the dearth of data from inclinations near 65°. Subsequently Moore[10] has determined orbits from Hewitt camera, kinetheodolite, visual and radar observations, and has re-analysed the change in inclination to obtain

$$10^9\bar{C}^{0,1}_{15} = 13.4 \pm 6.2 \qquad 10^9\bar{S}^{0,1}_{15} = 0.7 \pm 13.3 \quad . \tag{11}$$

We have used these instead of the previous values: however, as before[1], it was necessary to double the standard deviation of the $\bar{C}$ coefficient.

5.4 1971-54A, SESP-1

The satellite 1971-54A, at inclination 90.2°, suffered significant perturbations due to 15th-order resonance for more than 5 years, and analysis of 269 orbits, between November 1972 and January 1978, yielded excellent values[7] of lumped harmonics of order 15 and 30.

In this analysis, however, the perturbations due to earth tides were ignored, and Dr Philip Moore of the University of Aston has pointed out that the inclination could be significantly affected by such perturbations, because they are also near-resonant. The earth-tide perturbation, calculated using the equations and models adopted by Moore and Holland[11], is shown in Fig 1. The variation may be approximated as a linear increase, with maximum error 0.0005°, which is less than the errors in the observational values of inclination. This linear increase is at a rate of $(5.6 \pm 1.1) \times 10^{-6}$ deg/day, where an error of 20% has been assigned to cover the neglect of ocean tides and uncertainty in the Love number.

The previous analysis of 1971-54A was made using the THROE computer program and assuming that the variation in inclination, after removal of known perturbations, was due only to resonance. To discover whether the earth-tide perturbation was significant, the analysis was repeated with a linear term included. As a result of this change, the measure of fit ε was reduced from 0.52 to 0.48, and the value determined for the linear term gave a rate of change of inclination of $(4.5 \pm 0.5) \times 10^{-6}$ deg/day, which agrees with the pre-calculated value, $(5.6 \pm 1.1) \times 10^{-6}$ deg/day. The new fitting is, therefore, to be preferred.

The values for the lumped harmonics which emerge from the new fitting are as follows:

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$$\left.\begin{aligned} 10^9\bar{C}^{0,1}_{15} &= -16.05 \pm 0.21 & 10^9\bar{S}^{0,1}_{15} &= -6.90 \pm 0.21 \\ 10^9\bar{C}^{0,2}_{30} &= -9.81 \pm 0.58 & 10^9\bar{S}^{0,2}_{30} &= 9.00 \pm 0.75 \end{aligned}\right\} \quad . \qquad (12)$$

Although $\bar{C}^{0,1}_{15}$ is little changed from its previous value (-16.40 ± 0.24), there is a considerable change in $\bar{S}^{0,1}_{15}$ (previously -5.37 ± 0.15). The new values of $\bar{C}^{0,2}_{30}$ and $\bar{S}^{0,2}_{30}$ differ from the previous values (-8.2 and 11.1) by about 1.3 times the sum of the standard deviations.

5.5 1966-63A, OV1-8

The previous results for this satellite, at inclination 144.2°, have now been superseded by an improved analysis[12], in which 32 orbits were determined from observations and the effects of solar radiation pressure were taken into account. The new values,

$$10^9\bar{C}^{0,1}_{15} = 36900 \pm 9700 \qquad 10^9\bar{S}^{0,1}_{15} = 12200 \pm 4700 \quad , \qquad (13)$$

have much lower standard deviations than the previous values, and also fit the solutions much better.

6 THE 15TH-ORDER EQUATIONS

6.1 Odd degree

Each of the 24 satellites gives values for lumped harmonics of odd degree, so that in the notation of equation (6) we have 24 equations of the form

$$\bar{C}^{0,1}_{15} = \bar{C}_{15,15} + Q^{0,1}_{17}\bar{C}_{17,15} + Q^{0,1}_{19}\bar{C}_{19,15} + \cdots \quad , \qquad (14)$$

with similar equations for the S coefficients. The values of $\bar{C}^{0,1}_{15}$ and $\bar{S}^{0,1}_{15}$ for the 24 satellites are listed in Table 1, with the values of a, e, i and $\bar{F}_{15,15,7}$. The values of the Q coefficients up to degree 41, calculated with the RAE computer program PROF, appear in Table 2 (on page 22).

Following the method that proved successful before[1], we also add constraint equations of the form

$$\bar{C}_{\ell,15} = 0 \pm 10^{-5}/\ell^2 \quad , \qquad (15)$$

with similar equations for S, for $\ell = 15, 17, 19, \ldots$ up to the highest ℓ evaluated. These equations express the expectation[5] that the order of magnitude of the individual coefficients of degree ℓ is $10^{-5}/\ell^2$, for $15 \leq \ell < 40$, as is confirmed in a general way by the Goddard Earth Model 10C (Ref 13).

Thus in solving for N odd-degree harmonics, we have $24 + N$ equations for C, and another $24 + N$ equations for S.

6.2 Even degree

Lumped harmonics of even degree are obtained from 17 of the 24 satellites, so that we have 17 equations of the form

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Table 1

Values of lumped harmonics $(\bar{C},\bar{S})^{0,1}_{15}$ for the 24 satellites

No.	Satellite	i (deg)	Semi-major axis km	e	$10^9\bar{C}^{0,1}_{15}$	$10^9\bar{S}^{0,1}_{15}$	$\bar{F}_{15,15,7}$	$10^9\bar{F}_{15,15,7}\bar{C}^{0,1}_{15}$	$10^9\bar{F}_{15,15,7}\bar{S}^{0,1}_{15}$
1	65-09A	31.76	6857	0.007	30980 ± 1960	13540 ± 960	136.3×10^{-6}	4.22 ± 0.27	1.85 ± 0.13
2	69-68B	32.97	6857	0.004	20340 ± 750	6280 ± 910	216.0×10^{-6}	4.39 ± 0.16	1.36 ± 0.20
3	64-84A	37.80	6860	0.042	560 ± 580	−2000 ± 1450	1.111×10^{-3}	0.62 ± 0.64	−2.22 ± 1.61
4	79-82A	43.60	6862	0.001	−467 ± 34	−767 ± 106	5.576×10^{-3}	−2.55 ± 0.19	−4.28 ± 0.59
5	71-30B	46.36	6869	0.011	−596 ± 308†	−869 ± 188†	0.01074	−6.40 ± 3.31	−9.33 ± 2.02
6	74-34A	50.64	6872	0.002	−430.2 ± 10.0	−320.9 ± 8.3	0.02620	−11.27 ± 0.26	−8.41 ± 0.22
7	71-58B	51.05	6874	0.011	−354 ± 94*	−248 ± 45	0.02834	−10.03 ± 2.66	−7.03 ± 1.28
8	62-15A	53.82	6876	0.022	−370 ± 14	−114 ± 31	0.04657	−17.23 ± 0.65	−5.31 ± 1.44
9	65-53B	56.04	6879	0.003	−233.4 ± 3.3	−103 ± 34†	0.06681	−15.59 ± 0.22	−6.88 ± 2.27
10	63-24B	58.20	6883	0.002	−110.6 ± 5.6	−41.6 ± 4.5	0.09207	−10.18 ± 0.52	−3.83 ± 0.41
11	70-87A	62.92	6888	0.007	−5.4 ± 3.6	−31.4 ± 11.2†	0.1683	−0.91 ± 0.61	−5.28 ± 1.88
12	77-12B	65.49	6894	0.029	13.4 ± 12.4*	0.7 ± 13.3	0.2217	2.97 ± 2.75	0.16 ± 2.95
13	71-106A	65.70	6895	0.045	−36 ± 42*	9 ± 17	0.2264	−8.15 ± 9.51	2.04 ± 3.85
14	71-10B	65.83	6893	0.002	−0.7 ± 4.1	2.4 ± 3.9	0.2293	−0.16 ± 0.94	0.55 ± 0.89
15	71-18B	69.84	6900	0.040	−37 ± 24†	10 ± 6	0.3261	−12.07 ± 7.83	3.26 ± 1.96
16	70-111A	74.00	6905	0.001	−26.0 ± 1.0	−5.2 ± 1.3	0.4312	−11.21 ± 0.43	−2.24 ± 0.56
17	71-13B	74.05	6905	0.002	−24.6 ± 1.3	−6.1 ± 1.0	0.4324	−10.63 ± 0.56	−2.64 ± 0.43
18	77-95B	75.82	6908	0.029	−22.5 ± 5.1	−3.0 ± 5.4	0.4745	−10.68 ± 2.42	−1.42 ± 2.56
19	67-42A	80.17	6918	0.007	−23.1 ± 1.6	−8.6 ± 1.3	0.5594	−12.92 ± 0.90	−4.81 ± 0.73
20	70-19A	81.16	6916	0.005	−21.0 ± 1.6	−1.1 ± 5.2†	0.5736	−12.05 ± 0.92	−0.63 ± 2.98
21	67-73A	85.98	6925	0.025	−13.9 ± 2.3	−6.4 ± 3.3	0.6076	−8.45 ± 1.40	−3.89 ± 2.01
22	71-54A	90.21	6930	0.002	−16.05 ± 0.21	−6.90 ± 0.21	0.5855	−9.40 ± 0.12	−4.04 ± 0.12
23	64-52B	98.68	6945	0.023	−28.3 ± 2.0	1.5 ± 8.0†	0.4247	−12.02 ± 0.85	0.64 ± 3.40
24	66-63A	144.16	7009	0.003	36900 ± 9700	12200 ± 4700	61.95×10^{-6}	2.28 ± 0.60	0.76 ± 0.29

Key: * Standard deviation × 2.

† Standard deviation × 4.

$$\bar{C}_{15}^{1,0} = \bar{C}_{16,15} + Q_{18}^{1,0}\bar{C}_{18,15} + Q_{20}^{1,0}\bar{C}_{20,15} + \ldots , \qquad (16)$$

and another 17 of the form

$$\bar{C}_{15}^{-1,2} = \bar{C}_{16,15} + Q_{18}^{-1,2}\bar{C}_{18,15} + Q_{20}^{-1,2}\bar{C}_{20,15} + \ldots . \qquad (17)$$

Thus there are 34 equations for even-degree C coefficients, and another 34 for the S coefficients. With the constraint equations (15), we have (34 + N) equations when solving for N even-degree harmonics. The 17 pairs of values of $\bar{C}_{15}^{1,0}$, and $\bar{S}_{15}^{1,0}$, and the 17 pairs of $\bar{C}_{15}^{-1,2}$ and $\bar{S}_{15}^{-1,2}$, are given in Table 3, with the corresponding $\bar{F}$ factors. The Q coefficients up to degree 42 are given in Tables 4 and 5 (pages 23 and 24).

7 SOLUTIONS FOR INDIVIDUAL HARMONIC COEFFICIENTS OF 15TH ORDER

7.1 Method

The method of solution is described in Ref 1. Basically it is a least-squares solution with the option of relaxing the standard deviations of ill-fitting points when necessary, to keep the weighted residuals below a chosen level, usually about 1.4.

7.2 Odd-degree harmonics (ℓ = 15,17,19...)

When the 24 equations (14) and N constraint equations (15) were solved for N coefficients, with $7 \leq N \leq 11$, the measure of fit ε took the following values:

N	7	8	9	10	11
C equations	3.53	1.92	0.99	0.97	0.85
S equations	1.20	1.14	1.12	0.84	0.83

As usual, ε^2 is the sum of squares of weighted residuals divided by the number of degrees of freedom, and the weighted residual is the residual for each lumped coefficient divided by the standard deviation for that coefficient as given in Table 1.

At least 10 harmonics - up to ℓ = 33 - are needed because the Q values are large for the low-inclination satellites, as Table 2 shows. Increasing N from 10 to 11 reduces ε by 12% for the C equations, though only marginally for the S equations. Increases of N beyond 11 reduce ε by only 1% or less. So the 11-harmonic solution has been chosen as the most satisfactory.

The values of the odd-degree $\bar{C}$ and $\bar{S}$ coefficients given by the 11-harmonic solution are listed in Table 6. The standard deviations are on average 11% lower than in our previous solution[1]. This is a substantial improvement, in view of the fact that only one new satellite was added to the existing 23, and only four of the others were revised, mostly in a minor manner. The mean difference between the new values and the old is 1.2 × (the sum of the standard deviations), so the new solution is significantly different from the old. For example, $10^9\bar{C}_{15,15}$ changes from -22.7 to -20.7, and $10^9\bar{S}_{15,15}$ from -7.4 to -6.5.

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Table 3

Values of lumped harmonics $(\bar{C},\bar{S})_{15}^{1,0}$ and $(\bar{C},\bar{S})_{15}^{-1,2}$ for the 17 satellites

Satellite	i (deg)	$10^9\bar{C}_{15}^{1,0}$	$10^9\bar{C}_{15}^{-1,2}$	$10^9\bar{S}_{15}^{1,0}$	$10^9\bar{S}_{15}^{-1,2}$	$\bar{F}_{16,15,8}$	$\bar{F}_{16,15,7}$	$10^9\bar{F}_{16,15,8}\bar{C}_{15}^{1,0}$	$10^9\bar{F}_{16,15,7}\bar{C}_{15}^{-1,2}$	$10^9\bar{F}_{16,15,8}\bar{S}_{15}^{1,0}$	$10^9\bar{F}_{16,15,7}\bar{S}_{15}^{-1,2}$
79-82A	43.60	-860 ± 150		-1930 ± 160		9.277×10^{-3}		-8.0 ± 1.4		-17.9 ± 1.5	
			-234 ± 34		185 ± 268†		42.65×10^{-3}		-10.0 ± 1.5		7.9 ± 11.4
71-30B	46.36	86 ± 700*		-2020 ± 1120†		18.22×10^{-3}		1.6 ± 12.8		-36.8 ± 20.4	
			-6 ± 98		175 ± 74		72.35×10^{-3}		-0.4 ± 7.1		12.7 ± 5.4
74-34A	50.64	-211.2 ± 24.9		128.4 ± 20.2		45.17×10^{-3}		-9.5 ± 1.1		5.8 ± 0.9	
			-3.0 ± 8.6		63.5 ± 3.4		0.1440		-0.4 ± 1.2		9.1 ± 0.5
71-58B	51.05	-466 ± 232†		253 ± 120*		48.86×10^{-3}		-22.8 ± 11.3		12.4 ± 5.9	
			-50 ± 46*		45 ± 14		0.1526		-7.6 ± 7.0		6.9 ± 2.1
62-15A	53.82	-76 ± 18		172 ± 76*		80.16×10^{-3}		-6.1 ± 1.4		13.8 ± 6.1	
			151 ± 60†		11 ± 34		0.2180		32.9 ± 13.1		2.4 ± 7.4
65-53B	56.04	18 ± 17		57 ± 23		0.1141		2.1 ± 1.9		6.5 ± 2.6	
			106.9 ± 8.7		2.4 ± 8.1		0.2780		29.7 ± 2.4		0.7 ± 2.3
63-24B	58.20	59.3 ± 10.2		26.8 ± 7.4		0.1551		9.2 ± 1.6		4.2 ± 1.1	
			101.5 ± 6.5		12.9 ± 4.1		0.3395		34.5 ± 2.2		4.4 ± 1.4
71-106A	65.70	51 ± 24		-55 ± 70∇		0.3455		17.6 ± 8.3		-19.0 ± 24.2	
			-67 ± 40†		-19 ± 24		0.5129		-34.4 ± 20.5		-9.7 ± 12.3
71-10B	65.83	35.1 ± 23.4*		-13.8 ± 22.0*		0.3491		12.3 ± 8.2		-4.8 ± 7.7	
			-20.0 ± 10.7		-18.9 ± 10.3		0.5145		-10.3 ± 5.5		-9.7 ± 5.3
70-111A	74.00	-18.0 ± 3.3		-44 ± 25∇		0.5145		-9.3 ± 1.7		-22.6 ± 12.9	
			-46.5 ± 2.7		-40.5 ± 4.0		0.4402		-20.5 ± 1.2		-17.8 ± 1.8
71-13B	74.05	-19.8 ± 1.8		-24.8 ± 0.7		0.5149		-10.2 ± 0.9		-12.8 ± 0.4	
			-45.5 ± 2.0		-35.2 ± 1.0		0.4386		-20.0 ± 0.9		-15.4 ± 0.4
77-95B	75.82	-3 ± 26*		-4 ± 22*		0.5200		-1.6 ± 13.5		-2.1 ± 11.4	
			-63 ± 15		-46 ± 18		0.3732		-23.5 ± 5.6		-17.2 ± 6.7
67-42A	80.17	-54.7 ± 6.4*		-37.2 ± 2.6		0.4617		-25.3 ± 3.0		-17.2 ± 1.2	
			-131 ± 21*		-97 ± 18*		0.1552		-20.3 ± 3.3		-15.1 ± 2.8
70-19A	81.16	-26 ± 41∇		-15 ± 20†		0.4337		-11.3 ± 17.8		-6.5 ± 8.7	
			-128 ± 29		-130 ± 37		0.09806		-12.6 ± 2.8		-12.7 ± 3.6
67-73A	85.98	-87 ± 38		84 ± 120*		0.2281		-19.8 ± 8.7		19.2 ± 27.4	
			-122 ± 65		-175 ± 86		-0.1827		22.3 ± 11.9		32.0 ± 15.7
71-54A	90.21	-92 ± 48		-170 ± 112*		-0.01237		1.1 ± 0.6		2.1 ± 1.4	
			-62.9 ± 2.6		-53.4 ± 1.6		-0.3833		24.1 ± 1.0		20.5 ± 0.6
64-52B	98.68	-88 ± 28†		-37 ± 8		-0.4287		37.7 ± 12.0		15.9 ± 3.4	
			-3 ± 10*		-34 ± 11		-0.5139		1.5 ± 5.1		17.5 ± 5.7

Key: * Standard deviation × 2. † Standard deviation × 4.
∇ Standard deviation × 10.

Table 6

Values of odd-degree $\bar{C}_{\ell,15}$ and $\bar{S}_{\ell,15}$ given by the 11-harmonic solution

ℓ	$10^9\bar{C}_{\ell,15}$	$10^9\bar{S}_{\ell,15}$
15	-20.7 ± 0.5	-6.5 ± 0.4
17	7.3 ± 0.8	2.4 ± 0.7
19	-16.2 ± 0.7	-13.7 ± 0.6
21	17.9 ± 0.6	10.8 ± 0.9
23	20.6 ± 1.3	2.0 ± 1.3
25	-6.0 ± 1.8	1.1 ± 2.1
27	-4.6 ± 1.3	9.8 ± 2.4
29	-6.9 ± 1.4	-4.0 ± 1.4
31	18.4 ± 2.4	-4.9 ± 3.4
33	-1.1 ± 2.8	-12.4 ± 3.6
35	-10.5 ± 4.0	4.2 ± 4.4

The weighted residuals in the 24 satellite equations and the 11 constraint equations are given in Table 7. The weighted residuals are expressed relative to the standard deviations given in Table 1, some of which include factors of increase, as indicated by the footnotes. These factors, either 2 or 4, were chosen to keep all the weighted residuals for the individual satellites less than 1.4.

Table 7

Weighted residuals in the 35 equations for odd-degree harmonics, from the 11-harmonic solution

Satellite equations			Constraint equations		
Satellite	$\bar{C}^{0,1}_{15}$	$\bar{S}^{0,1}_{15}$	ℓ	$\bar{C}_{\ell,15}$	$\bar{S}_{\ell,15}$
65-09A	0.08	0.31	15	0.47	0.15
69-68B	0.25	0.27	17	-0.21	-0.07
64-84A	-0.19	-0.83	19	0.59	0.49
79-82A	-0.01	0.34	21	-0.79	-0.48
71-30B	-0.88	-0.99	23	-1.09	-0.11
74-34A	0.24	-0.15	25	0.37	-0.07
71-58B	0.85	0.86	27	0.34	-0.71
62-15A	-0.80	0.30	29	0.58	0.33
65-53B	-0.24	-1.15	31	-1.77	0.47
63-24B	0.96	0.27	33	0.13	1.35
70-87A	-0.64	-1.34	35	1.28	-0.52
77-12B	1.18	0.05			
71-106A	-0.82	0.47			
71-10B	0.33	0.18			
71-18B	-0.90	0.61			
70-111A	-0.58	0.19			
71-13B	0.67	-0.53			
77-95B	0.91	1.05			
67-42A	-0.10	-0.37			
70-19A	0.02	1.22			
67-73A	0.13	0.09			
71-54A	-0.01	-0.01			
64-52B	-0.19	1.17			
66-63A	-1.17	-1.39			

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The weighted residuals for the individual harmonic coefficients, which are of course relative to the arbitrary standard deviation, $10^{-5}/\ell^2$, have an average value of 0.55; so the constraint seems to be at about the right level. By relaxing the constraint to, say, $2 \times 10^{-5}/\ell^2$, a nominally more accurate solution can be obtained, but with greater danger of oscillatory excursions in the values. The numerical values of the 15th-order harmonics of degree 15-35, as given in Table 6 (and Table 8), are plotted in Fig 2, where the values of $\pm 10^{-5}/\ell^2$ are also indicated. Fig 2 seems to confirm that the constraint is reasonable.

The values of $\bar{F}_{15,15,7}\bar{C}^{1,0}_{15}$ and $\bar{F}_{15,15,7}\bar{S}^{1,0}_{15}$ from Table 1 are plotted against inclination in Fig 3, and the variation given by the 11-harmonic solution is shown by the unbroken lines. The fitting of the points seems to be excellent - it is faithful and has no unlikely-looking oscillations.

7.3 Even-degree harmonics (ℓ = 16,18,20,...)

When the 34 equations of the form (16) or (17) and the N constraint equations were solved for N coefficients, for $6 \leq N \leq 10$, the measure of fit ε took the following values:

N	6	7	8	9	10
C equations	1.67	1.09	1.09	1.04	0.99
S equations	2.03	1.04	0.93	0.92	0.90

Increasing N from 9 to 10 significantly reduces ε for both C and S equations, but there was no appreciable improvement in going beyond N = 10. So the 10-harmonic solution was chosen, because

(a) it is probably the best, on the basis of ε, and

(b) it fits in with the 11-harmonic solution for coefficients of odd degree.

The values of $\bar{C}$ and $\bar{S}$ coefficients given by the 10-harmonic solution are listed in Table 8. The only change from the previous solution was the addition of 1971-10B: the standard deviations are on average 5% lower than in the previous solution and the values of the coefficients are very similar.

Table 8

Values of even-degree $\bar{C}_{\ell,15}$ and $\bar{S}_{\ell,15}$ given by the 10-harmonic solution

ℓ	$10^9\bar{C}_{\ell,15}$	$10^9\bar{S}_{\ell,15}$
16	-12.1 ± 2.3	-21.7 ± 1.5
18	-42.4 ± 1.7	-22.3 ± 1.1
20	-23.5 ± 2.0	-6.0 ± 1.5
22	23.9 ± 2.0	10.2 ± 1.6
24	0.4 ± 3.6	-22.1 ± 3.2
26	-14.3 ± 5.5	14.3 ± 5.3
28	-15.2 ± 6.3	-8.4 ± 6.3
30	-3.1 ± 6.7	-15.7 ± 6.2
32	9.2 ± 5.9	3.0 ± 5.0

The pattern of residuals is almost the same as before, so no Table is given. The weighted residuals for 1971-10B, expressed relative to the standard deviations given in Table 3, are -1.24 for $\bar{C}_{15}^{1,0}$, 0.08 for $\bar{C}_{15}^{-1,2}$, 1.06 for $\bar{S}_{15}^{1,0}$ and 0.39 for $\bar{S}_{15}^{-1,2}$. None of the weighted residuals in the 34 satellite equations exceeded 1.4, relative to the standard deviations of Table 3.

The values of $\bar{F}_{16,15,8}\bar{C}_{15}^{1,0}$ and $\bar{F}_{16,15,7}\bar{C}_{15}^{-1,2}$ are plotted against inclination in Fig 4; and Fig 5 is a similar diagram for the S coefficients. The variations given by the 10-harmonic solution are shown by the unbroken lines in Figs 4 and 5. The only peculiarity that strikes the eye in Figs 4 and 5 is the peak in $\overline{FC}_{15}^{1,0}$ near $i = 63^\circ$ in Fig 4, which is higher than might be expected. This arises because the fittings of equations (16) and (17) are simultaneous and the Q coefficients for $\bar{C}_{15}^{-1,2}$ at $i \simeq 58^\circ$ are similar to the Q coefficients for $\bar{C}_{15}^{1,0}$ at $i \simeq 64^\circ$: since the $\overline{FC}_{15}^{-1,2}$ curve is pulled upwards by the observational point at $i = 58.2^\circ$, there is a corresponding peak in $\overline{FC}_{15}^{1,0}$ near 64°.

Although the 10-harmonic solution is needed to match the 11-harmonic solution of Table 6, it is obvious that the higher-degree coefficients in Table 8 are poorly determined. So it would be useful to have a solution with a smaller number of coefficients and lower standard deviations. But it is found that none of the other solutions (N = 6, 7, 8 or 9) fulfils these requirements. The standard deviations are only very slightly smaller, and it seems that the advantage of having smaller numbers of coefficients is balanced by the errors due to neglecting the fairly large high-degree harmonics. If a smaller number of coefficients, n, is required, the best plan seems to be to take the first n coefficients in Table 8.

8 COMPARISON OF 15TH-ORDER COEFFICIENTS WITH THOSE OF COMPREHENSIVE GEOPOTENTIAL MODELS

The comparisons of gravity models made by Klokočník and Pospíšilová[14] suggest that only the recent models are sufficiently reliable for a useful comparison with our new values. However, several recent geopotential models, such as Rapp's 1981 model[15] and the European GRIM 3[16], have utilized our previous values of 15th-order harmonics, so that comparisons are fruitless. One recent model which is believed to be independent of our results is the Goddard Earth Model 10B[13], and our values are compared with GEM 10B up to degree 23 in Table 9.

The object of this comparison is to assess the accuracy of GEM 10B on the assumptions (a) that our standard deviations are realistic and (b) that GEM 10B is less accurate than our values. The mean numerical difference between the 18 GEM values in Table 9 and the corresponding values in our solutions is 3.6×10^{-9}. This crude but effective method of comparison suggests that the accuracy of the GEM 10B values is about 3 or 4×10^{-9}. The mean standard deviation of our values is 1.2×10^{-9}, sufficiently smaller to justify the direct comparison.

For the coefficients of degree 24-35, listed in Table 10, the mean difference between the 24 GEM values and the corresponding values in our solution is 8.1×10^{-9}. Since eight of our values have standard deviations greater than 5×10^{-9}, this comparison

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Table 9

Comparison of our values with GEM 10B up to degree 23

ℓ	$10^9\bar{C}_{\ell,15}$		$10^9\bar{S}_{\ell,15}$	
	GEM 10B	Tables 6 and 8	GEM 10B	Tables 6 and 8
15	-19.7	-20.7 ± 0.5	-6.4	-6.5 ± 0.4
16	-14.4	-12.1 ± 2.3	-27.8	-21.7 ± 1.5
17	2.5	7.3 ± 0.8	4.8	2.4 ± 0.7
18	-48.3	-42.4 ± 1.7	-18.6	-22.3 ± 1.1
19	-20.6	-16.2 ± 0.7	-15.3	-13.7 ± 0.6
20	-23.9	-23.5 ± 2.0	4.8	-6.0 ± 1.5
21	16.2	17.9 ± 0.6	9.5	10.8 ± 0.9
22	24.1	23.9 ± 2.0	-1.3	10.2 ± 1.6
23	15.4	20.6 ± 1.3	4.1	2.0 ± 1.3

may not give a fair impression. If we exclude these eight values, the mean difference between the 16 remaining GEM values and the corresponding values in our solution is 7.0×10^{-9}, while the mean standard deviation of our 16 values is 3.0×10^{-9}. Any comparison between uncertain values is open to criticism, and all that can be said is that the accuracy of GEM 10B seems to deteriorate as the degree increases beyond 23 - a conclusion which will cause no surprise. Since only two of the 24 GEM 10B coefficients have numerical values exceeding 7×10^{-9}, the GEM 10B values may be largely indeterminate for degree 24-35.

Table 10

Comparison of our values with GEM 10B for degree 24-35

ℓ	$10^9\bar{C}_{\ell,15}$		$10^9\bar{S}_{\ell,15}$	
	GEM 10B	Tables 6 and 8	GEM 10B	Tables 6 and 8
24	3.1	0.4 ± 3.6	-5.1	-22.1 ± 3.2
25	-1.6	-6.0 ± 1.8	-10.2	1.1 ± 2.1
26	4.6	-14.3 ± 5.5	1.2	14.3 ± 5.3
27	0.6	-4.6 ± 1.3	-1.1	9.8 ± 2.4
28	-6.8	-15.2 ± 6.3	-1.9	-8.4 ± 6.3
29	-7.0	-6.9 ± 1.4	-6.1	-4.0 ± 1.4
30	-5.7	-3.1 ± 6.7	-0.4	-15.7 ± 6.2
31	-0.3	18.4 ± 2.4	2.3	-4.9 ± 3.4
32	1.4	9.2 ± 5.9	-2.5	3.0 ± 5.0
33	1.0	-1.1 ± 2.8	3.3	-12.4 ± 3.6
34	1.9	10.4 ± 5.8	0.6	5.3 ± 4.9
35	-13.7	-10.5 ± 4.0	2.7	4.2 ± 4.4

Fig 3 to 5 offer graphical comparisons of GEM 10B (broken lines) with our solutions (full lines). Fig 3 shows good agreement. Thus the 15th-order coefficients of odd degree in GEM 10B, taken as a whole, provide good values of lumped harmonics over the whole range of inclination, even though the individual coefficients are ill defined for high degree. In Figs 4 and 5 the agreement between GEM 10B and our graphs is not quite

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so good, but the form of the curves is similar for both solutions, apart from a divergence with $\bar{S}^{1,0}_{15}$ on Fig 5 at inclinations less than 48°. For the even-degree values, too, GEM 10B therefore provides reasonably realistic values for lumped 15th-order harmonics over quite a wide range of inclinations, whatever the deficiencies in the values of the individual harmonics.

This direct comparison of actual values gives a more favourable impression of the accuracy of GEM 10B than the statistical tests applied by Lambeck and Coleman[17].

9 THE 30TH-ORDER EQUATIONS AND THEIR SOLUTIONS

9.1 The equations available

Of the 24 orbits analysed at 15th-order resonance, eight have given useful values for lumped harmonics of 30th order and even degree (seven previously used, plus 1971-10B), while only one has provided values for lumped harmonics of odd degree.

Therefore only the even-degree coefficients can be evaluated, and for these the lumped harmonics are of the form

$$\bar{C}^{0,2}_{30} = \bar{C}_{30,30} + Q^{0,2}_{32}\bar{C}_{32,30} + Q^{0,2}_{34}\bar{C}_{34,30} + \ldots , \qquad (18)$$

with the same equations for S on replacing C by S throughout. To these equations we add constraint equations of the form

$$\bar{C}_{\ell,30} = 0 \pm 10^{-5}/\ell^2 , \qquad (19)$$

and similarly for S, for $\ell = 30, 32, 34, \ldots$ up to the highest ℓ evaluated. Thus, in solving for N even-degree 30th-order harmonics, we have $8 + N$ equations for C and also for S.

The values of $\bar{C}^{0,2}_{30}$ and $\bar{S}^{0,2}_{30}$ obtained from the eight satellites are given in Table 11, together with values of $\bar{F}_{30,30,14}$, $\bar{F}\bar{C}$ and $\bar{F}\bar{S}$, which give a better idea of the relative contributions of the 30th-order terms in equation (7), where the factor $2(R/a)^{15}$ has a numerical value of approximately 0.6. Table 12 lists the Q coefficients.

Table 11

Values of even-degree lumped harmonics $(\bar{C},\bar{S})^{0,2}_{30}$ for the eight satellites

Satellite	i (deg)	$10^9\bar{C}^{0,2}_{30}$	$10^9\bar{S}^{0,2}_{30}$	$\bar{F}_{30,30,14}$	$10^9\bar{F}_{30,30,14}\bar{C}^{0,2}_{30}$	$10^9\bar{F}_{30,30,14}\bar{S}^{0,2}_{30}$
1974-34A	50.64	597 ± 558	679 ± 651	0.000952	0.57 ± 0.53	0.65 ± 0.62
1963-24B	58.20	46 ± 106	253 ± 88	0.01176	0.54 ± 1.25	2.98 ± 1.03
1971-10B	65.83	-54 ± 27	59 ± 40	0.07292	-3.94 ± 1.97	4.30 ± 2.92
1970-111A	74.00	19.2 ± 4.9	4.1 ± 4.4	0.2579	4.95 ± 1.26	1.06 ± 1.13
1971-13B	74.05	27.1 ± 11.0*	6.0 ± 3.3	0.2594	7.03 ± 2.85	1.56 ± 0.86
1967-42A	80.17	-9.1 ± 9.2*	-5.0 ± 11.0*	0.4340	-3.95 ± 3.99	-2.17 ± 4.77
1971-54A	90.21	-9.81 ± 0.58	9.00 ± 0.75	0.4755	-4.66 ± 0.28	4.28 ± 0.36
1964-52B	98.68	22.8 ± 7.9	38.0 ± 41.2†	0.2502	5.70 ± 1.98	9.51 ± 10.31

* standard deviation × 2

† standard deviation × 4

Table 12

Values of $Q_{32}^{0,2}$, $Q_{34}^{0,2}$, ..., $Q_{42}^{0,2}$ for the eight satellites

Satellite	$Q_{32}^{0,2}$	$Q_{34}^{0,2}$	$Q_{36}^{0,2}$	$Q_{38}^{0,2}$	$Q_{40}^{0,2}$	$Q_{42}^{0,2}$
1974-34A	-11.9	52.9	-131.8	200.3	-171.5	34.5
1963-24B	-7.61	19.01	-20.11	2.71	11.18	-2.92
1971-10B	-3.971	3.166	1.636	-1.323	-1.607	-0.149
1970-111A	-1.121	-0.806	-0.128	0.319	0.433	0.314
1971-13B	-1.107	-0.807	-0.140	0.308	0.429	0.317
1967-42A	0.155	-0.161	-0.281	-0.295	-0.252	-0.184
1971-54A	0.430	0.213	0.100	0.036	-0.000	-0.020
1964-52B	-1.113	-0.847	-0.281	0.127	0.313	0.333

9.2 Solutions for individual harmonic coefficients of 30th order and even degree

When the $8 + N$ equations were solved for N coefficients, with $2 \leq N \leq 6$, the values of ε were as follows:

N	2	3	4	5	6
C equations	1.75	1.38	0.98	0.96	0.87
S equations	1.35	0.71	0.70	0.70	0.69

The best value of N is not at all clear from these results. For the S equations it is enough to take $N = 3$, giving harmonics up to $\ell = 34$, but only because the coefficients for $\ell = 36$, 38 and 40 are small (all being less than half their standard deviation, as the solutions show). *A priori*, however, it would be expected that six harmonics, up to $\ell = 40$, should be evaluated, because Table 12 shows that the largest Q coefficients for 1974-34A are Q_{38} and Q_{40}. For the C equations there is an advantage in increasing N from 4 to 6, since ε decreases by 11%. A value of N higher than 6 cannot be accepted, because there are only seven satellites distinct in inclination.

In these circumstances it seems appropriate to give two solutions, for $N = 6$ and for a lower value. This lower value is chosen as $N = 4$, because the C equations are fitted much better at $N = 4$ than at $N = 3$. These two solutions are given in Table 13, with the GEM 10B values (which cease at $\ell = 36$). The standard deviations of the new 6-harmonic solution are on average 29% lower than in our previous solution, so the inclusion of 1971-10B has brought about a great improvement. Previously, only the values of the $\ell = 30$ coefficients could be regarded as well defined. In the new solution the values for $\ell = 30$, 32 and 34 seem reliable, with good consistency between the 4-harmonic and 6-harmonic solutions.

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Table 13

Values of $\bar{C}_{\ell,30}$ and $\bar{S}_{\ell,30}$ of even degree given by our 6- and 4-harmonic solutions, and by GEM 10B

ℓ	$10^9\bar{C}_{\ell,30}$			$10^9\bar{S}_{\ell,30}$		
	6-harmonic	4-harmonic	GEM 10B	6-harmonic	4-harmonic	GEM 10B
30	-3.4 ± 1.1	-2.6 ± 1.0	-5.2	8.3 ± 0.9	8.6 ± 0.6	11.1
32	-7.9 ± 2.4	-8.3 ± 2.7	-0.6	-2.9 ± 2.2	-3.3 ± 2.1	-0.2
34	-11.8 ± 2.9	-12.8 ± 3.2	-11.9	9.0 ± 2.6	8.8 ± 2.5	1.2
36	-6.4 ± 3.9	-8.9 ± 3.2	-3.9	-0.6 ± 3.3	-1.3 ± 2.6	-0.9
38	5.0 ± 3.9			1.5 ± 3.2		
40	4.4 ± 3.2			1.3 ± 2.7		

Table 14 gives the weighted residuals in the 6-harmonic solution relative to the standard deviations in Table 11, four of which are increased to ensure that the weighted residuals do not exceed 1.4.

Table 14

Weighted residuals in the 14 equations used in evaluating the 6-harmonic solution for 30th order and even degree

Satellite equations			Constraint equations		
Satellite	$\bar{C}^{0,2}_{30}$	$\bar{S}^{0,2}_{30}$	Degree ℓ	$\bar{C}_{\ell,30}$	$\bar{S}_{\ell,30}$
1974-34A	0.05	0.02	30	0.31	-0.75
1963-24B	0.20	0.24	32	0.80	0.30
1971-10B	-0.75	0.40	34	1.35	-1.03
1970-111A	0.00	-0.30	36	0.84	0.08
1971-13B	0.73	0.20	38	-0.73	-0.21
1967-42A	-0.61	-0.98	40	-0.71	-0.21
1971-54A	-0.04	0.10			
1964-52B	0.47	0.81			

The values of $\bar{F}_{30,30,14}\bar{C}^{0,2}_{30}$ and $\bar{F}_{30,30,14}\bar{S}^{0,2}_{30}$ are plotted against inclination in Fig 6 with the variations given by our 6-harmonic solution (unbroken lines) and our 4-harmonic solution (dot-dash lines). With the $\bar{S}$-coefficients there is nothing to choose between the two fittings, but for the $\bar{C}$-coefficients the 6-harmonic solution gives a noticeably better fit and is also less extreme in its variations.

9.3 Comparison with comprehensive geopotential models

The GEM 10B values can appropriately be compared with our 4-harmonic solution, and the agreement is surprisingly good. The mean numerical difference between the eight GEM values and the corresponding values in our solution is 3.7×10^{-9}. Since the mean

standard deviation of our values is 2.2×10^{-9}, it is possible that the 30th-order harmonics of even degree in GEM 10B may have errors as low as 3×10^{-9}.

In Fig 6 the variations of the lumped harmonics from our solutions are compared with the variation given by GEM 10B, shown by the broken lines. For the $\bar{C}$-coefficients the GEM 10B curve is of the same form as our solution, and all our eight values are fitted to within 2.8 standard deviations. For the $\bar{S}$-coefficients the agreement is not so good but the values are fairly small and are again fitted to within 2.8 standard deviations. All in all, GEM 10B emerges well from the comparisons.

For the 30th-order harmonics, GRIM 3 is independent of our values, and Rapp[15] uses only our previous (30,30) coefficient. So comparisons are legitimate, and in Table 15 our 4-harmonic solution is set beside GRIM 3 and our 6-harmonic solution beside Rapp's model.

Table 15

Values of even-degree 30th-order coefficients given by our 4-harmonic solution, GRIM 3, our 6-harmonic solution and Rapp[15]

ℓ	$10^9\bar{C}_{\ell,30}$				$10^9\bar{S}_{\ell,30}$			
	4-harm.	GRIM 3	6-harm.	Rapp	4-harm.	GRIM 3	6-harm.	Rapp
30	-2.6	-2.7	-3.4	(-3.3)	8.6	2.9	8.3	(7.5)
32	-8.3	-2.1	-7.9	-6.7	-3.3	-2.6	-2.9	0.5
34	-12.8	-24.2	-11.8	-22.9	8.8	-7.7	9.0	-0.6
36	-8.9	-11.4	-6.4	-6.0	-1.3	1.9	-0.6	4.8
38			5.0	0.5			1.5	2.5
40			4.4	1.2			1.3	-1.3

The mean difference between the eight GRIM values and ours is 5.8×10^{-9}, while the mean difference between the ten independent Rapp values and ours is 4.2×10^{-9}. Thus Rapp's model agrees with our values almost as well as GEM 10B, while for GRIM 3 the agreement is not so good. However, the mean difference is less, 4.3×10^{-9}, in the recent GRIM3-L1 model[18].

10 DISCUSSION

10.1 The 15th-order harmonics

Our previous evaluation of 15th order harmonics[1] had one unsatisfactory feature: there was only one accurate orbit analysis for inclinations between 59° and 73°, and that orbit (1970-87A) gave values only for odd-degree harmonics. Furthermore the analysis of this satellite was of questionable reliability, because it was based on an inadequate number of orbits (23), all at times *after* resonance.

This deficiency in the previous data has now been put right by the addition of 1971-10B at inclination 65.9°, and the resulting coverage of the inclination range is now

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quite satisfactory (see Figs 3 to 5), although further accurate orbit analyses, particularly at inclinations between 59° and 73°, would improve the accuracy and reliability of the results.

The curves of odd-degree lumped harmonics in Fig 3 differ significantly from their previous form at inclinations between about 60° and 70°. For the $\bar{C}^{0,1}_{15}$ coefficients, it appears that the previously available values near 65° and 70° inclination were in error by somewhat more than their standard deviations, and the same applies to the S coefficient obtained from 1970-87A at 62.9°.

For the even-degree lumped harmonics, shown in Figs 4 and 5, the previous curves were very poorly defined between 60° and 70° inclination, and changes were to be expected. As it turns out, however, the new values for 1971-10B fit the old curves quite well, and no changes of any importance occur.

The new values of the 15th-order coefficients (Tables 6 and 8) should be more reliable than the previous set, because the gap previously existing has been filled by 1971-10B and because improved values of odd-degree lumped harmonics have been obtained from other satellites, particularly 1977-12B, 1971-54A and 1966-63A. The standard deviations of the new solutions are on average smaller than for the old, by 11% for the odd-degree harmonics and by 5% for the even-degree harmonics. The average standard deviation for the coefficients of degree 15, 16, 17, ..., 23 is 1.2×10^{-9}, equivalent to about 1 cm in geoid height.

Figs 3 to 5 show that the Goddard Earth Model 10B provides quite realistic values for the lumped harmonics over the whole range of inclinations, though it is probable that the GEM 10B values for coefficients of degree higher than 24 are almost indeterminate.

10.2 The 30th-order harmonics

Our previous evaluation of 30th-order harmonics of even degree was a first attempt, and the only values securely established were those of the 30th-degree coefficients. Previously there were no results at all for inclinations between 59° and 73°. Again the data from 1971-10B has filled this gap, and the new solution (Table 13) establishes good values for $\ell = 32$ and 34 as well as $\ell = 30$. The average standard deviation for these three pairs of coefficients is nearly 40% lower than in the previous solution, being 2.0×10^{-9}, equivalent to about 1½ cm in geoid height. Also the 4-harmonic and 6-harmonic solutions agree well, thus suggesting that the $\ell = 36$ values may be reliably determined.

The variations of the lumped harmonics given by the 4- and 6-harmonic solutions are shown in Fig 6 with values from GEM 10B for comparison. The 6-harmonic solution is seen to be preferable for the $\bar{C}$-coefficients; and GEM 10B agrees surprisingly well with our solutions.

11 CONCLUSIONS

A new set of values has been derived for the individual 15th-order harmonic coefficients in the geopotential of degree 15, 16, 17, ..., 35. See Tables 6 and 8. These values should be more reliable because the gap in inclination near 65° has now been filled by the results from 1971-10B. The new solution also has lower standard deviations,

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and for degree 15, 16, ..., 23 these correspond to an accuracy of about 1 cm in the undulation in geoid height produced by 15th-order harmonics.

The analysis of 1971-10B has also provided values of lumped 30th-order harmonics at an inclination previously quite unrepresented. In the new solution (Table 13) three pairs of coefficients, of degree 30, 32 and 34, are evaluated with an accuracy equivalent to 1½ cm in geoid height, and there is an improvement in accuracy of nearly 40%.

The results have been used to test the Goddard Earth Model 10B, and the model emerges rather well from the comparison, with an indication of errors of about 3 or 4×10^{-9} for coefficients of order 15 and degree 15-23, and also for those of order 30 and degree 30, 32, 34 and 36.

Acknowledgments

We thank Dr Philip Moore of the University of Aston for supplying his results on 1977-12B before publication, and for pointing out how the analysis of 1971-54A could be improved.

Table 2

VALUES OF $Q_{17}^{0,1}$, $Q_{19}^{0,1}$ ... $Q_{41}^{0,1}$ FOR THE 24 SATELLITES

Satellite	$Q_{17}^{0,1}$	$Q_{19}^{0,1}$	$Q_{21}^{0,1}$	$Q_{23}^{0,1}$	$Q_{25}^{0,1}$	$Q_{27}^{0,1}$	$Q_{29}^{0,1}$	$Q_{31}^{0,1}$	$Q_{33}^{0,1}$	$Q_{35}^{0,1}$	$Q_{37}^{0,1}$	$Q_{39}^{0,1}$	$Q_{41}^{0,1}$
65-09A	-12.0	59.0	-186.0	426.0	-755.0	1058.0	-1166.0	963.0	-490.0	-48.0	397.0	-416.0	178.0
69-68B	-11.5	55.1	-166.3	362.0	-600.6	771.1	-750.2	497.8	-113.0	-208.8	309.9	-183.2	-30.4
64-84A	-10.0	40.6	-99.8	167.1	-194.4	143.8	-34.3	-61.6	79.6	-25.2	-35.0	43.7	-6.6
79-82A	-8.21	25.64	-44.67	44.93	-18.22	-13.86	21.25	-2.92	-12.92	7.67	5.90	-7.42	-1.71
71-30B	-7.32	19.59	-27.19	17.57	3.13	-12.98	3.87	7.22	-4.74	-3.77	4.01	1.94	-3.06
74-34A	-5.97	11.94	-9.87	-0.53	6.22	-1.10	-3.92	0.91	2.65	-0.46	-1.86	0.09	1.30
71-58B	-5.84	11.30	-8.71	-1.24	5.74	-0.34	-3.70	0.29	2.50	0.02	-1.71	-0.27	1.14
62-15A	-5.003	7.539	-2.913	-3.325	2.152	2.155	-1.147	-1.639	0.384	1.218	0.110	-0.802	-0.359
65-53B	-4.351	5.103	-0.257	-2.867	-0.045	1.804	0.533	-1.012	-0.748	0.370	0.677	0.066	-0.436
63-24B	-3.740	3.202	1.066	-1.745	-1.071	0.721	1.005	0.013	-0.665	-0.381	0.231	0.401	0.090
70-87A	-2.509	0.496	1.350	0.340	-0.615	-0.634	-0.073	0.361	0.343	0.045	-0.194	-0.201	-0.045
77-12B	-1.905	-0.278	0.819	0.693	0.039	-0.403	-0.385	-0.094	0.166	0.226	0.113	-0.040	-0.118
77-106A	-1.858	-0.323	0.770	0.697	0.082	-0.368	-0.386	-0.123	0.138	0.219	0.127	-0.020	-0.107
71-10B	-1.830	-0.350	0.741	0.701	0.107	-0.348	-0.387	-0.140	0.121	0.215	0.136	-0.007	-0.101
71-18B	-1.014	-0.759	-0.093	0.341	0.409	0.231	-0.003	-0.155	-0.180	-0.110	-0.011	0.062	0.083
70-111A	-0.331	-0.587	-0.454	-0.200	0.027	0.160	0.192	0.151	0.075	0.002	-0.049	-0.068	-0.061
71-13B	-0.323	-0.583	-0.455	-0.204	0.021	0.156	0.190	0.152	0.078	0.005	-0.046	-0.067	-0.061
77-95B	-0.087	-0.411	-0.429	-0.307	-0.144	-0.002	0.090	0.126	0.119	0.084	0.040	0.000	-0.027
67-42A	0.346	0.059	-0.097	-0.171	-0.192	-0.177	-0.144	-0.102	-0.061	-0.026	0.002	0.020	0.030
70-19A	0.416	0.153	-0.003	-0.092	-0.136	-0.147	-0.138	-0.116	-0.089	-0.060	-0.035	-0.013	0.002
67-73A	0.588	0.420	0.313	0.236	0.178	0.132	0.097	0.069	0.047	0.031	0.018	0.009	0.002
71-54A	0.513	0.327	0.221	0.154	0.110	0.079	0.058	0.042	0.031	0.023	0.017	0.012	0.009
64-52B	-0.268	-0.515	-0.466	-0.324	-0.175	-0.054	0.029	0.076	0.095	0.093	0.079	0.059	0.038
66-63A	-14.0	77.0	-266.0	651.0	-1198.0	1693.0	-1824.0	1398.0	-559.0	-254.0	613.0	-435.0	6.0

Table 4

VALUES OF $Q_{18}^{1,0}$, $Q_{20}^{1,0}$ ... $Q_{42}^{1,0}$ FOR THE 17 SATELLITES

Satellite	$Q_{18}^{1,0}$	$Q_{20}^{1,0}$	$Q_{22}^{1,0}$	$Q_{24}^{1,0}$	$Q_{26}^{1,0}$	$Q_{28}^{1,0}$	$Q_{30}^{1,0}$	$Q_{32}^{1,0}$	$Q_{34}^{1,0}$	$Q_{36}^{1,0}$	$Q_{38}^{1,0}$	$Q_{40}^{1,0}$	$Q_{42}^{1,0}$
79-82A	-6.2	18.9	-35.0	39.2	-20.0	-10.4	23.4	-7.7	-13.4	12.8	4.4	-11.7	1.0
71-30B	-5.46	14.35	-21.14	15.44	1.54	-12.56	5.77	6.96	-6.96	-3.20	6.16	1.13	-4.93
74-34A	-4.42	8.59	-7.43	-0.39	5.59	-1.48	-3.92	1.54	2.93	-1.13	-2.30	0.66	1.84
71-58B	-4.32	8.11	-6.51	-1.02	5.19	-0.70	-3.79	0.81	2.88	-0.49	-2.24	0.11	1.73
62-15A	-3.67	5.27	-1.95	-2.84	1.97	2.01	-1.35	-1.71	0.69	1.48	-0.11	-1.16	-0.31
65-53B	-3.16	3.44	0.10	-2.42	-0.08	1.78	0.48	-1.19	-0.79	0.59	0.86	-0.05	-0.68
63-24B	-2.67	2.02	1.07	-1.40	-1.04	0.71	1.06	-0.04	-0.82	-0.42	0.38	0.55	0.05
71-106A	-1.160	-0.534	0.506	0.698	0.180	-0.357	-0.461	-0.174	0.174	0.308	0.185	-0.039	-0.181
71-10B	-1.138	-0.551	0.478	0.698	0.206	-0.333	-0.461	-0.196	0.152	0.303	0.199	-0.020	-0.172
70-111A	0.134	-0.404	-0.525	-0.358	-0.085	0.148	0.264	0.254	0.157	0.030	-0.075	-0.129	-0.128
71-13B	0.141	-0.399	-0.524	-0.362	-0.092	0.142	0.261	0.255	0.161	0.036	-0.070	-0.127	-0.129
77-95B	0.355	-0.170	-0.428	-0.442	-0.300	-0.101	0.077	0.187	0.216	0.178	0.102	0.019	-0.049
67-42A	0.780	0.457	0.152	-0.084	-0.237	-0.308	-0.312	-0.267	-0.195	-0.113	-0.036	0.026	0.069
70-19A	0.857	0.594	0.320	0.081	-0.100	-0.217	-0.275	-0.282	-0.254	-0.202	-0.140	-0.077	-0.022
67-73A	1.112	1.105	1.041	0.946	0.834	0.716	0.599	0.487	0.385	0.294	0.214	0.147	0.091
71-54A	1.177	1.250	1.270	1.256	1.220	1.169	1.108	1.040	0.970	0.899	0.828	0.759	0.693
64-52B	0.861	0.605	0.339	0.106	-0.071	-0.187	-0.247	-0.260	-0.239	-0.195	-0.140	-0.084	-0.034

Table 5

VALUES OF $Q_{18}^{-1,2}$, $Q_{20}^{-1,2}$ ... $Q_{42}^{-1,2}$ FOR THE 17 SATELLITES

Satellite	$Q_{18}^{-1,2}$	$Q_{20}^{-1,2}$	$Q_{22}^{-1,2}$	$Q_{24}^{-1,2}$	$Q_{26}^{-1,2}$	$Q_{28}^{-1,2}$	$Q_{30}^{-1,2}$	$Q_{32}^{-1,2}$	$Q_{34}^{-1,2}$	$Q_{36}^{-1,2}$	$Q_{38}^{-1,2}$	$Q_{40}^{-1,2}$	$Q_{42}^{-1,2}$
79-82A	-4.37	9.12	-10.09	3.79	4.61	-5.97	-0.29	4.80	-1.76	-3.11	2.41	1.70	-2.36
71-30B	-3.76	6.27	-4.42	-1.23	4.05	-0.76	-2.89	1.34	1.98	-1.37	-1.36	1.21	0.95
74-34A	-2.85	2.89	0.13	-2.18	0.28	1.66	-0.19	-1.30	0.00	1.01	0.17	-0.76	-0.29
71-58B	-2.76	2.63	0.35	-2.04	0.01	1.57	0.07	-1.21	-0.22	0.90	0.36	-0.62	-0.42
62-15A	-2.19	1.15	1.18	-0.89	-0.99	0.49	0.89	-0.11	-0.73	-0.19	0.49	0.35	-0.24
65-53B	-1.75	0.29	1.19	-0.02	-0.91	-0.32	0.57	0.52	-0.18	-0.49	-0.13	0.31	0.27
63-24B	-1.34	-0.27	0.88	0.51	-0.43	-0.62	-0.03	0.46	0.32	-0.14	-0.35	-0.13	0.18
71-106A	-0.104	-0.644	-0.467	0.031	0.384	0.378	0.109	-0.173	-0.274	-0.170	0.022	0.155	0.157
71-10B	-0.085	-0.635	-0.479	0.009	0.372	0.385	0.129	-0.156	-0.272	-0.183	0.005	0.146	0.161
70-111A	0.874	0.513	0.096	-0.245	-0.431	-0.446	-0.327	-0.140	0.044	0.173	0.223	0.197	0.121
71-13B	0.879	0.521	0.106	-0.237	-0.427	-0.448	-0.333	-0.149	0.036	0.168	0.221	0.199	0.126
77-95B	1.032	0.820	0.481	0.117	-0.188	-0.381	-0.443	-0.391	-0.263	-0.105	0.040	0.142	0.188
67-42A	1.407	1.634	1.687	1.581	1.351	1.038	0.689	0.347	0.045	-0.192	-0.353	-0.437	-0.453
70-19A	1.59	2.02	2.26	2.31	2.18	1.92	1.56	1.14	0.73	0.34	0.01	-0.24	-0.41
67-73A	0.988	0.879	0.741	0.598	0.461	0.334	0.222	0.125	0.044	-0.023	-0.075	-0.114	-0.141
71-54A	0.957	0.839	0.714	0.598	0.496	0.410	0.337	0.276	0.226	0.185	0.151	0.123	0.100
64-52B	0.236	-0.261	-0.468	-0.469	-0.357	-0.203	-0.054	0.063	0.138	0.172	0.172	0.147	0.109

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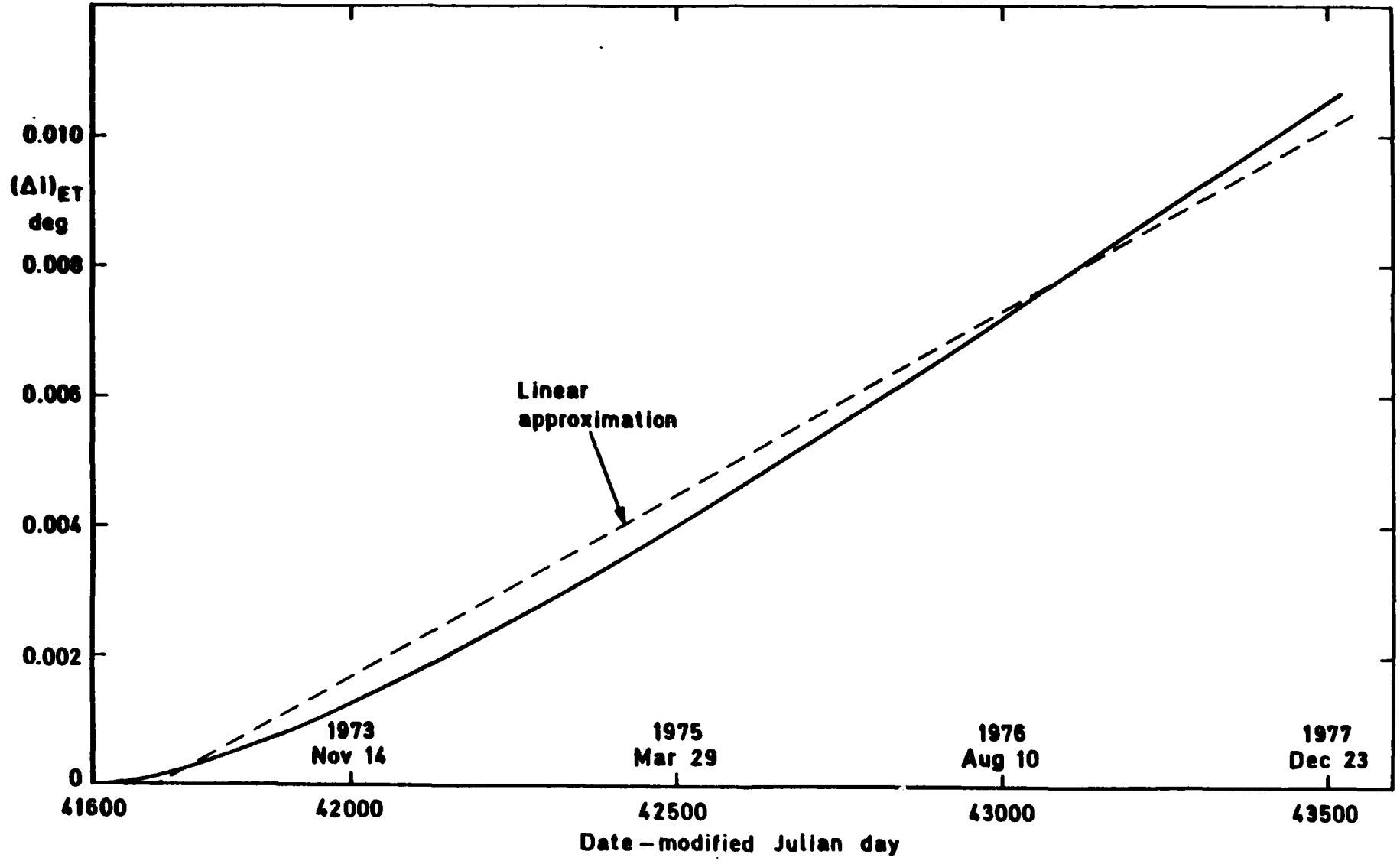

Fig 1 Calculated earth-tide perturbation to inclination, $(\Delta i)_{ET}$, for 1971-54A

Fig 1

Fig 2

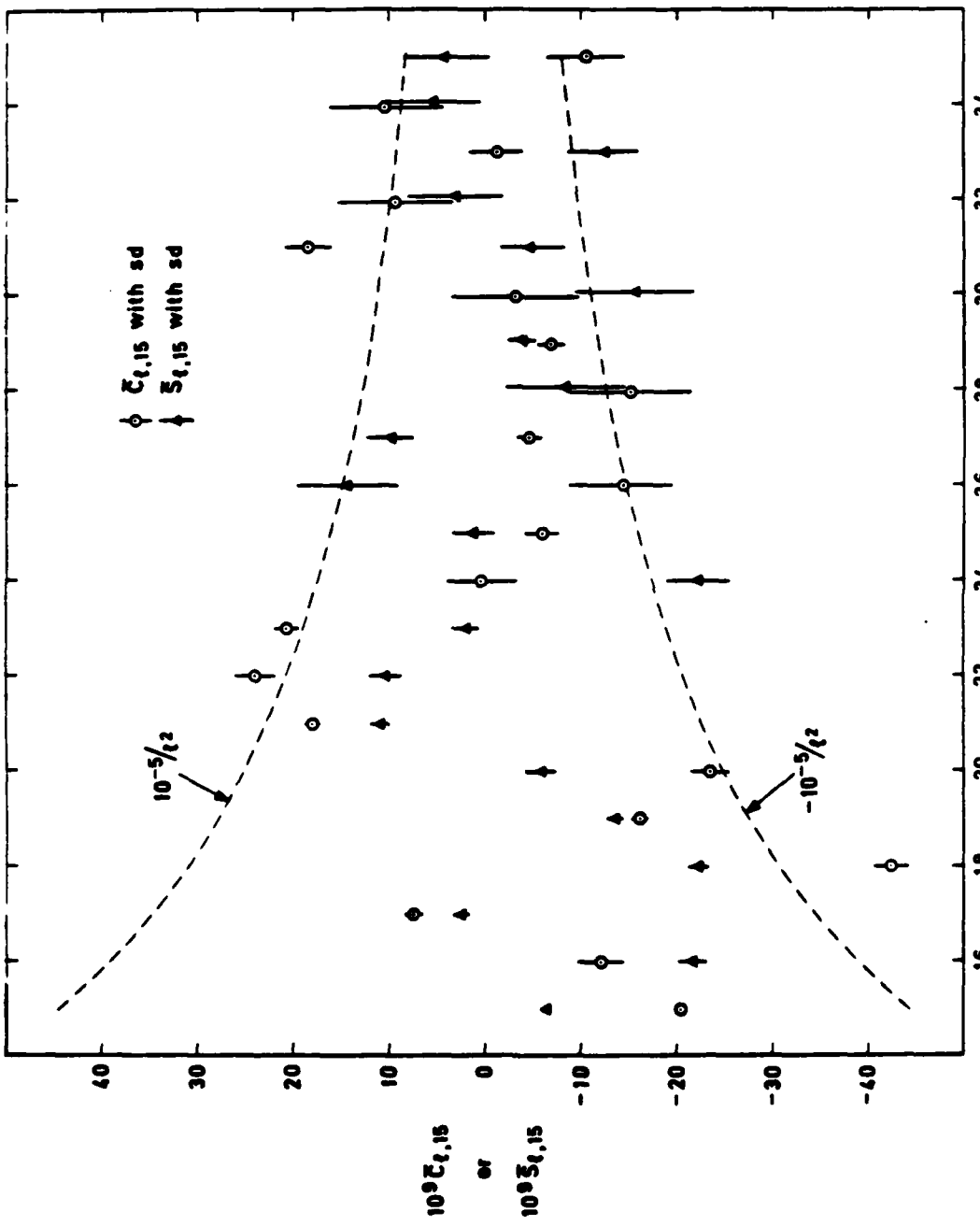

Fig 2 Values of $\bar{C}_{\ell,15}$ and $\bar{S}_{\ell,15}$ from Tables 6 and 8, to show their magnitudes relative to $\pm 10^{-5}/\ell^2$

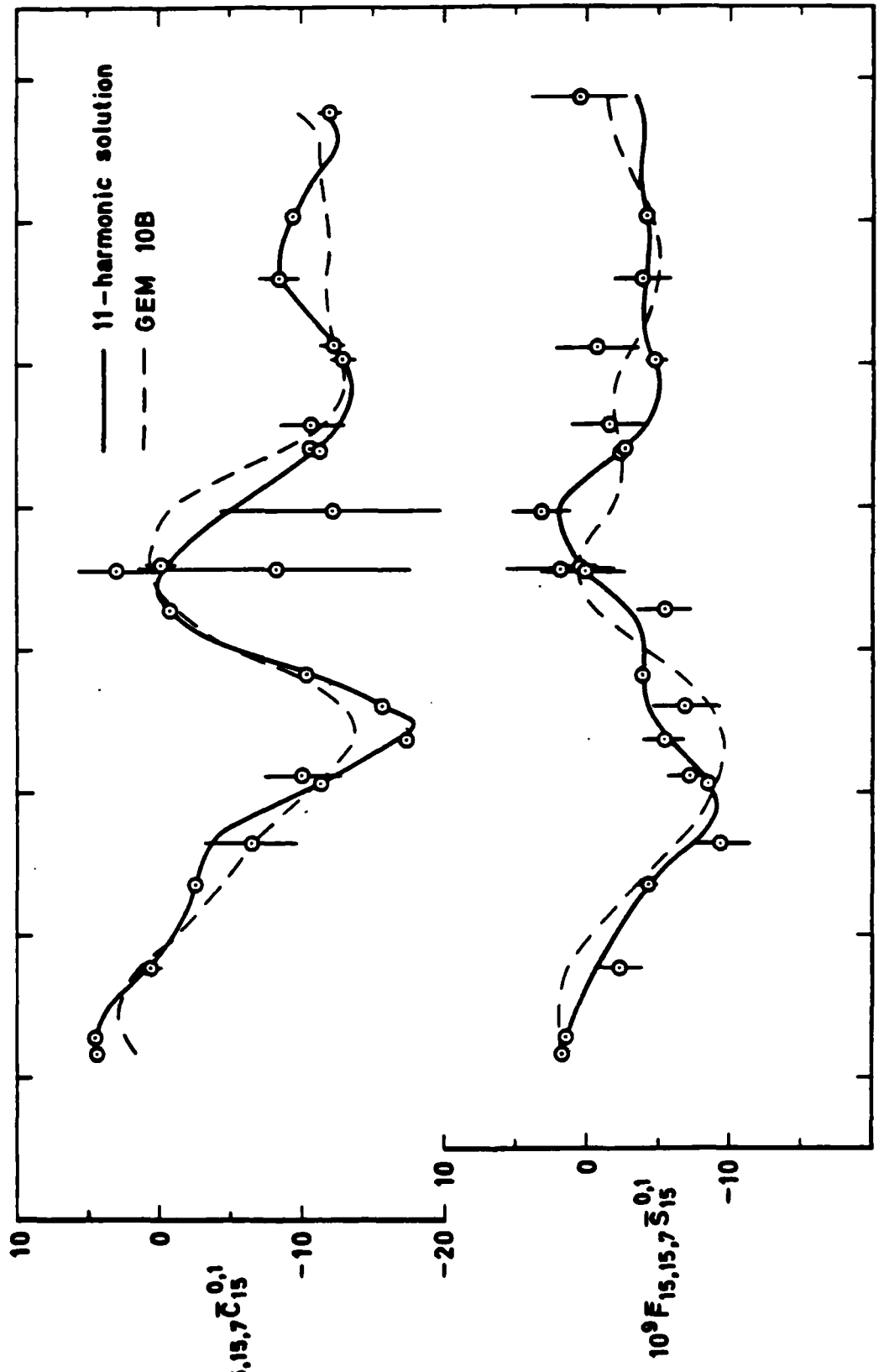

Fig 3 Values of $F_{15,15,7}\bar{C}^{0,1}_{15}$ and $F_{15,15,7}\bar{S}^{0,1}_{15}$ from Table 1, with the curves given by the 11-harmonic solution and GEM 10B

Fig 4

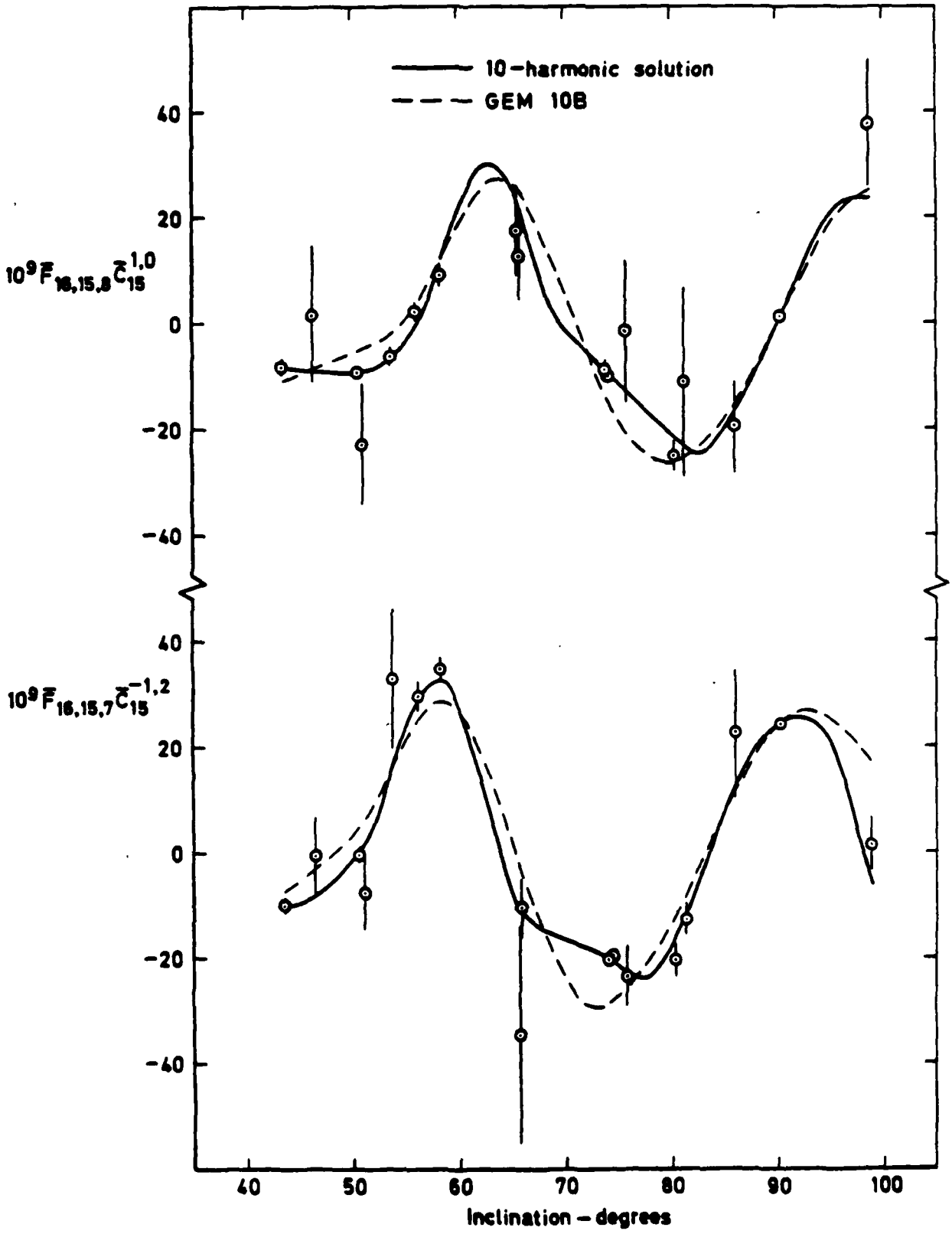

Fig 4 Values of $\bar{F}_{16,15,8}\bar{C}_{15}^{1,0}$ and $\bar{F}_{16,15,7}\bar{C}_{15}^{-1,2}$ from Table 3, with the curves given by the 10-harmonic solution and GEM 10B

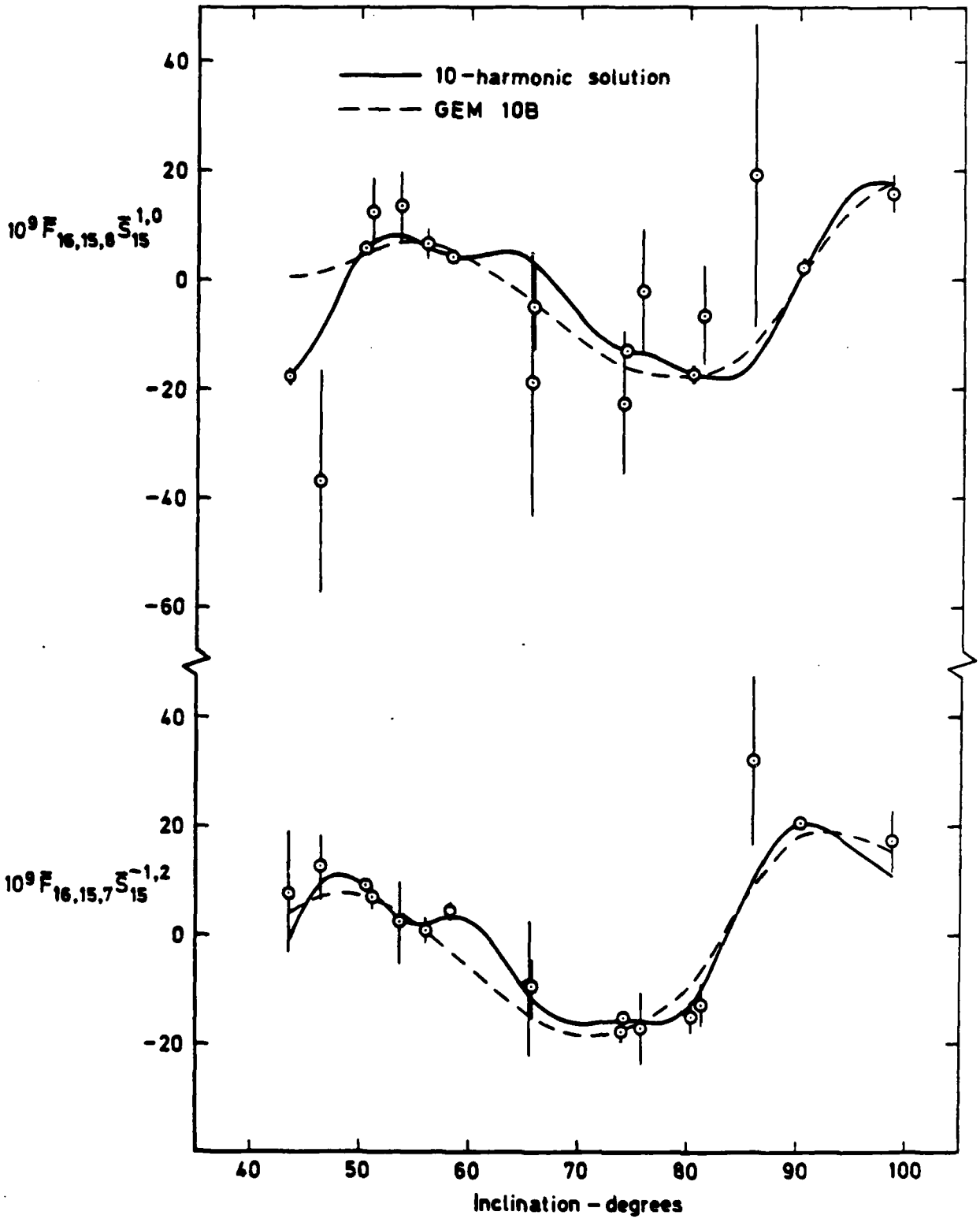

Fig 5 Values of $\bar{F}_{16,15,8}\bar{S}_{15}^{1,0}$ and $\bar{F}_{16,15,7}\bar{S}_{15}^{-1,2}$ from Table 3, with the curves given by the 10-harmonic solution and GEM 10B

Fig 6

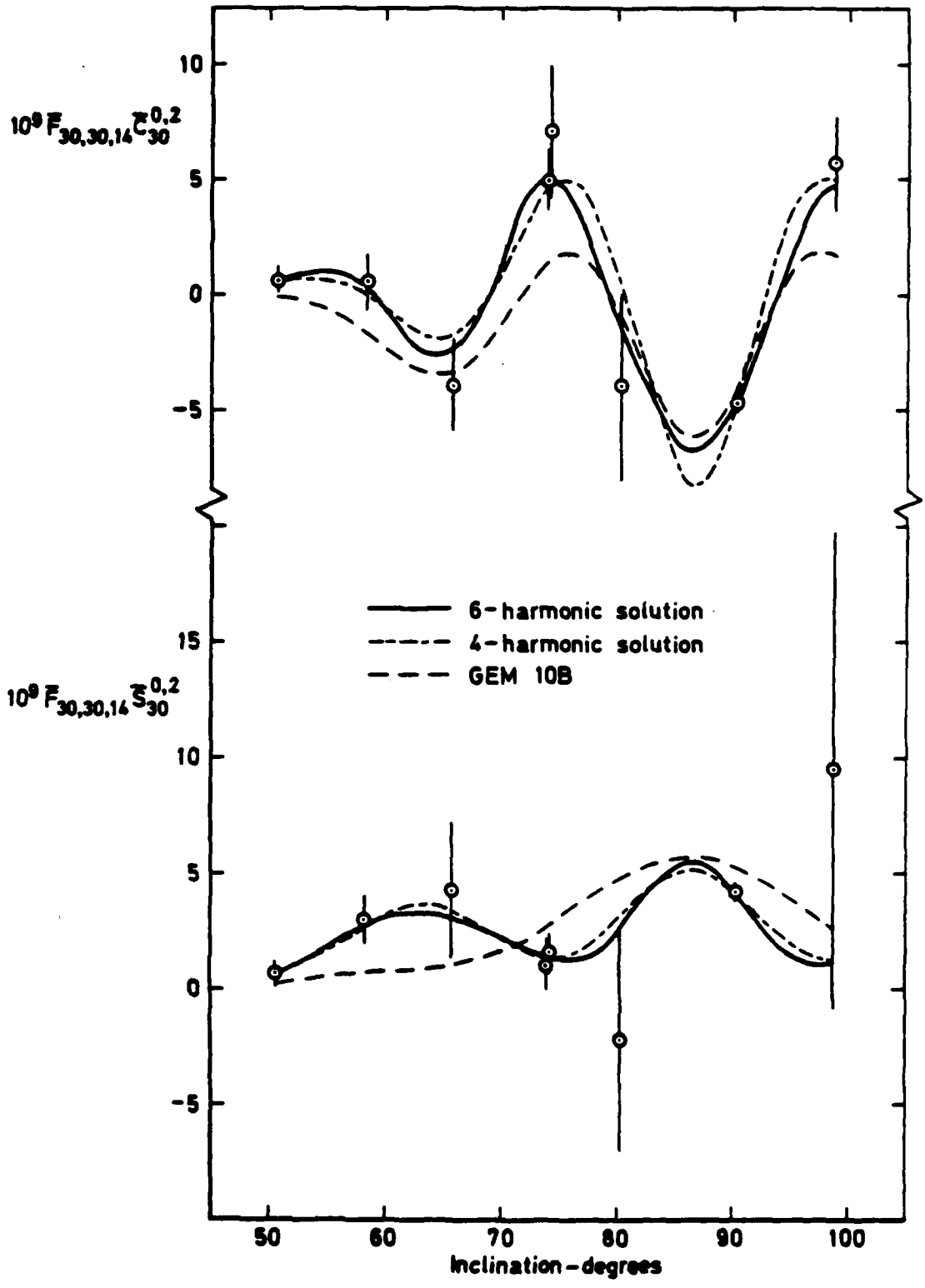

Fig 6 Values of $F_{30,30,14}C_{30}^{0,2}$ and $F_{30,30,14}S_{30}^{0,2}$ from Table 11, with the curves given by the 4- and 6-harmonic solutions and GEM 10B

REPORT DOCUMENTATION PAGE

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UNLIMITED

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17. Abstract

The analysis of variations in satellite orbits when they pass through 15th-order resonance (15 revolutions per day) yields values of lumped geopotential harmonics of order 15, and sometimes of order 30. The 15th-order lumped harmonics obtained from 24 such analyses over a wide range of orbital inclinations are used here to determine individual harmonic coefficients of order 15 and degree 15, 16, ... 35; and the 30th-order lumped harmonics (from eight of the analyses) are used to evaluate individual coefficients of order 30 and degree 30, 32, ... 40. The new values should be more accurate than any previously obtained. The accuracy of the 15th-order coefficients of degree 15, 16,... 23 is equivalent to 1 cm in geoid height, while the 30th-order coefficients of degree 30, 32 and 34 are determined with an accuracy which is equivalent to better than 2 cm in geoid height. The results are used to assess the accuracy of the Goddard Earth Model 10B.

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